smaller than one part in ten million, and found to be in good agreement with theory. This is a sensitive test of the theory, so there is continued interest in improved measurements of g.

The spin precession can be calculated by solving the spinor equation (5.35) for R or equation (5.38) for U. The latter equation not only looks simpler, it is sometimes easier to solve, and it determines both the velocity and the spin.

Its solution may be facilitated by putting it in the form

$$\dot{U} = \tfrac{1}{2}\left(\frac{eF}{mc}\right)U + \tfrac{1}{2}U\Big[\frac{e(g-2)}{2mc} i\mathbf{B}_0\Big], \tag{5.40}$$

where $i\mathbf{B}_0 = \widetilde{U}F_{-}U$. For constant F we see immediately that the solution can be put in the form

$$U = e^{(e/2mc)F\tau} L_0 R_0\,, \tag{5.41}$$

where $V_0 = cL_0^2 = \gamma_0(c + \mathbf{v}_0)$ is the initial 4-velocity and $R_0 = R_0(\tau)$ satisfies

$$\dot{R}_0 = \tfrac{1}{2}R_0\big[\frac{e(g-2)}{2mc}\, i\mathbf{B}_0\big]\,. \tag{5.42}$$

Note that (5.42) differs from (5.35) in describing only the "anomolous part" of the rotation. Likewise, $\mathbf{B}_0$ differes from $\mathbf{B}'$ in (5.27) by

$$i\mathbf{B}_0 = \langle\, \widetilde{R}_0 i\mathbf{B}' R_0 \,\rangle_2 \; = i\widetilde{R}_0 \left\{\mathbf{B} + \gamma\Big[\mathbf{E} + \frac{\gamma}{\gamma+1}\frac{\mathbf{v}}{c}\times\mathbf{B}\Big]\times\frac{\mathbf{v}}{c}\right\} R_0. \tag{5.43}$$

When $\mathbf{E} = 0$, both γ and the angle between $\mathbf{v}$ and $\mathbf{B}$ remain constant. Then if the initial velocity $\mathbf{v}_0$ is orthogonal to $\mathbf{B}$, the double cross product in (5.43) reduces to

$$\frac{\gamma_0^2}{\gamma_0+1}\,\frac{\mathbf{v}_0^2}{c}\,\mathbf{B} = (\gamma_0 + 1)\mathbf{B},$$

and $\mathbf{B}_0$ is constant if R commutes with $\mathbf{B}$. Subject to these initial conditions for motion in a pure magnetic field $F = i\mathbf{B}$, the solution (5.41) takes the form

$$U = e^{-\frac{1}{2}i\boldsymbol{\omega}t}\, L_0\, e^{-\frac{1}{2}i\boldsymbol{\omega}_0 t}\,, \tag{5.44}$$

where $t = \gamma_0\tau$ from (3.46),

$$\boldsymbol{\omega} = -\frac{e}{2mc\gamma_0}\,\mathbf{B} \tag{5.45a}$$

is the "relativistic frequency" and

$$\boldsymbol{\omega}_0 = \tfrac{1}{2}(g-2)\gamma_0\boldsymbol{\omega}\,. \tag{5.45b}$$

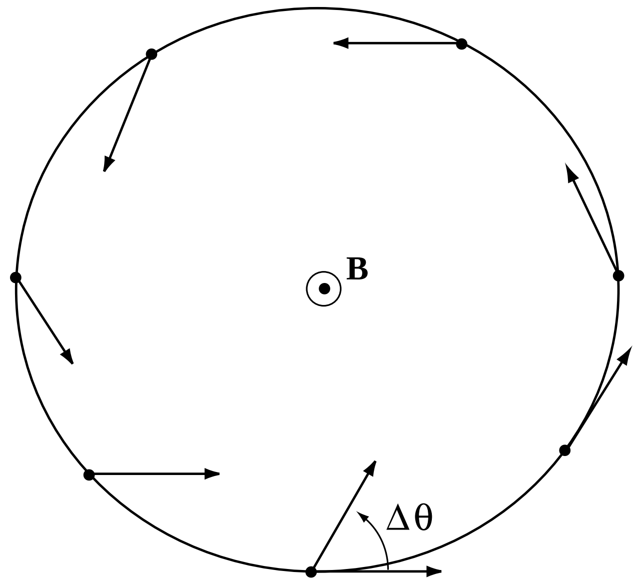

Figure 5.1. The spin vector, initially collinear with the velocity, is shown at successive points on a circular orbit in a magnetic field **B** (for $g>2$).

We can put (5.44) in the form $U = LR$, where

$$L = e^{-\frac{1}{2}i\boldsymbol{\omega}t}\, L_0\, e^{\frac{1}{2}i\boldsymbol{\omega}t} \tag{5.46a}$$

and

$$R = e^{-\frac{1}{2}i\boldsymbol{\omega}t}\, e^{\frac{1}{2}i\boldsymbol{\omega}_0 t}. \tag{5.46b}$$

Using (5.46b) in (5.34), we get the spin $\mathbf{s}$ as an explicit function of time.

This result has been used to measure g by measuring the spin precession. The first factor in (5.46b) gives the precession of the velocity with the cyclotron frequency $\boldsymbol{\omega}$. Therefore, the second factor describes a precession of the spin with respect to the velocity with frequency $\boldsymbol{\omega}_0$. For particles injected into the magnetic field with $\mathbf{v}$ perpendicular to $\mathbf{B}$, the orbit is circular with period $T = 2\pi/\omega$. Therefore, with each complete circuit the spin rotates through an angle

$$\Delta\theta = \omega_0 T = 2\pi\gamma_0\Big(\frac{g-2}{2}\Big) \tag{5.47}$$

(Fig. 5.1). The angle between the spin and velocity can be magnified by passing through many circuits.

For an arbitrary constant field F it is necessary to integrate (5.42) to evaluate R_0 in (5.41). The integration is straightforward, because the time dependence of $\mathbf{B}_0$ is determined by $\mathbf{v}(\tau)$ in (5.43), and that is already known from (3.67). To express the result in terms of lab time t, one needs (3.71) to convert from proper time τ. For the general case the result is too messy to be worth exhibiting here.

9-5 Exercises

(5.1) What sense does it make to compare directions of the magnetic field $\mathbf{B}$ and the particle velocity $\mathbf{v}$ in the LAB system with the direction of the particle's spin $\mathbf{s}$ in its rest system?